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ANALYTICAL MODELLING OF LOAD-DEFLECTION BEHAVIOR OF INTERVERTEB--ETC(U)

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ANALYTICAL MODELLING OF LOAD-DEFLECTION BEHAVIOR OF  
INTERVERTEBRAL DISCS SUBJECTED TO AXIAL COMPRESSION

by

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# ABSTRACT

The analytical modelling of creep response phenomena of intervertebral discs subjected to a constant axial compressive load is attempted by using Kelvin-solid models. A mathematical analysis scheme is proposed for unique model identification wherein exact parameter solutions are developed for the one-Kelvin-unit model, the three-parameter-solid model, and the two-Kelvin-unit model. In addition, a method is presented by which the associated Young's moduli and viscosity coefficients for an identified model are obtainable. Most importantly, unique parameter values are obtained for the three-parameter-solid by utilizing exact model parameter solutions on experimental strain,  $\epsilon(t)$ , data. This particular model is observed to yield theoretical strain,  $\epsilon(t)_{cal}$ , values that are within an average error of 3.48% of the experimentally measured values,  $\epsilon(t)_{exp}$ , for different intervertebral discs. Further, mechanical properties of the intervertebral discs are obtained by using the values of the three-parameter-solid model parameters to calculate the associated Young's moduli and viscosity coefficient. The appropriate applications, data limitations, and possible generalizations of this exact analysis scheme are fully discussed, along with suggestions for future investigatory efforts.

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## INTRODUCTION

An understanding of the mechanical properties and behavior of the spinal column is of general interest to researchers in many areas, ranging from problems associated with the man-machine environments in which man is subjected to external dynamic forces to those encountered in the selection of suitable materials for intervertebral disc replacement. Of particular interest to this investigation is the biomechanical modelling of intervertebral joint response to constant axial compressive loads, where an intervertebral joint is understood to be an intervertebral disc and adjoining vertebrae.

Nachemson (1960), Hirsch (1965), and Rolander (1966) performed experimental investigations to determine the load-deflection behavior of the intervertebral disc subjected to axial loading. More recently, Kazarian (1975) reported creep characteristics for intervertebral joints subjected to a constant axial stress, and Kazarian and Kaleps (1979) illustrated the determination of Young's moduli and a viscosity coefficient for a three-parameter-solid model based on this data.

A basic problem associated with the modelling of experimental compressive creep data is the identification of the most physically appropriate and analytically unique model. A lumped-parameter model consisting of a number of Kelvin solids in series may be sufficient in simulating experimentally observed creep phenomena. However, the number of Kelvin solids represented in such a model is generally dependent on the analytical and physical intuition of the investigator and his willingness and capability of identifying the values of unknown, independent model parameters via optimization techniques using the available experimental data.

One objective of this continuing research is to replace investigative intuition with analytical certitude in model identification. The exact mathematical solutions to a logical progression of mechanical models that are appropriate for experimental creep data analysis are presented and fully discussed. Due to the inherent mathematical complexity of the exact solution approach to model identification for a Kelvin chain consisting of more than three Kelvin solids, approximating solutions appear warranted for the generalized basic problem. The results

presented herein for three different analytical models can be easily programed on a digital computer to test experimentally measured creep data for unique model identification and to calculate the associated Young's moduli and viscosity coefficients for an identified model.

#### ON MODELLING COMPRESSIVE CREEP PHENOMENA WITH KELVIN SOLIDS

Typical experimental compressive creep data of strain  $\epsilon(t)$ , as illustrated by the dotted curve in Fig. 1 after Kazarian (1975), exhibits behavior which is similar to that of a Kelvin solid, where a one-Kelvin-unit solid is understood to be a spring and dashpot in parallel. Indeed, it is being suggested that a Kelvin chain, consisting of two or more Kelvin elements connected in series, might be sufficient to simulate compressive creep phenomena. The problem, of course, is to identify the minimum number of Kelvin units in the chain and to determine the independent parameter values associated with each Kelvin element, such that the predicted strain values of the Kelvin chain model are in good agreement with available experimental creep data.

The general solution for the strain behavior of an N-Kelvin-unit solid is given by Flügge (1975) as

$$\epsilon(t) = \sum_{i=1}^N A_i (1 - e^{-\lambda_i t}), \quad (1)$$

which would appear to be a logical test of the experimental compressive creep data. When a computerized optimization scheme employing Eq. 1 was applied to Kazarian's creep data for the T4 - T5 intervertebral joint, one value of N yielding a "reasonably" good fit occurred for  $N = 5$ . A plot of the calculated strain  $\epsilon(t)_{cal}$  values using a five-Kelvin-unit model is illustrated in Fig. 1 by a solid line. Clearly, after the first few minutes, the experimentally observed strain response is rather well predicted by the five-Kelvin-unit model. Although this result appears encouraging, further investigation revealed that the values used for the ten parameters ( $\lambda_i, A_i; i = 1, 2, 3, 4, 5$ ) were not unique. That is, these values could be altered slightly, or even drastically, with a resulting "reasonably good

fit" to the experimental creep response data. This alarming observation suggests that another model, perhaps simpler, might yield a unique fit to the experimental data. Once this modelling uncertainty was realized, no further effort was made to improve the optimization scheme used to identify the parameter values with respect to the experimental data. Further, it is suggested that the use of such a model, although instructive, is of questionable value. It is generally felt that "guessing" at parameter values, which is inherent in any computerized optimization scheme, should be avoided, if at all possible.

From this preliminary modelling attempt, the assumption that a Kelvin chain might be sufficient in simulating compressive creep response of intervertebral discs appeared reasonable. The immediate task was to uniquely identify the appropriate number of Kelvin elements in the Kelvin chain and the model parameter values by a more exact mathematical method. To this end, an analytical scheme is proposed wherein exact parameter solutions are derived for a few illustrative models. To be more specific, exact parameter solutions are developed for the one-Kelvin-unit model, the three-parameter-solid model, and the two-Kelvin-unit model, along with a discussion of their appropriate applications and limitations when used to analyze experimental strain data.

#### EXACT PARAMETER SOLUTIONS TO THE ONE-KELVIN-UNIT MODEL

The one-Kelvin-unit model, schematically represented in Fig. 2, is a linear model, since the strain behavior is directly proportional to the applied stress. Taking  $E_1$  and  $\eta_1$  as representing Young's modulus and the coefficient of viscosity, respectively, the total stress applied,  $\sigma$ , can be expressed by

$$\sigma = E_1 \epsilon + \eta_1 \dot{\epsilon} \equiv q_0 \epsilon - q_1 \dot{\epsilon}, \quad (2)$$

where the dot denotes time-differentiation and the second equality is just the standard form for the total stress.  $E_1$  and  $\eta_1$  are related to the unit elasticity,  $k_1$ , and damping coefficient,  $c_1$ , by the equations

$$E_1 = k_1 l / A \quad (3)$$

and  $\eta_1 = c_1 l / A, \quad (4)$

where  $l$  is the initial length and  $A$  is the effective cross-sectional area of the specimen. For a constant stress

$$\sigma \equiv \sigma_0, t > 0, \quad (5)$$

the solution for the strain,  $\epsilon(t)$ , to the constitutive equation, Eq. 2, under the condition of creep may be expressed by

$$\epsilon(t) = \sigma_0 J(t), \quad (6)$$

where  $J(t)$  is the creep compliance. According to Flügge (1975), the creep compliance for this model is given by

$$J(t) = (1/q_0)(1 - e^{-\lambda_1 t}), \quad (7)$$

where the parameter is

$$\lambda_1 = q_0/q_1. \quad (8)$$

Combining Eqs. 6 and 7, the solution for the strain behavior of the Kelvin solid can be represented in the general form

$$\epsilon(t) = A_1(1 - e^{-\lambda_1 t}), \quad (9)$$

where  $\lambda_1$  and  $A_1$  are independent parameters requiring determination for the complete specification of  $\epsilon(t)$ . At this point, using Eqs. 3, 6, 7, 8, and 9, it is easily shown that

$$E_1 = \sigma_0/A_1 \quad (10)$$

$$\text{and} \quad \eta_1 = \sigma_0/\lambda_1 A_1. \quad (11)$$

Clearly, Eqs. 10 and 11 can be utilized in determining Young's modulus,  $E_1$ , and the viscosity coefficient,  $\eta_1$ , once unique values for the parameters  $\lambda_1$  and  $A_1$  are obtained.

Assuming this simple model can be used to analyze observed compressive creep phenomena, then the experimental data for  $\epsilon(t)$  and  $t$  may be expressed in the form

$$\epsilon_1 = A_1(1 - e^{-\lambda_1 t_1}), \quad (12.1)$$

$$\epsilon_{i+1} = A_1(1 - e^{-\lambda_1 t_{i+1}}). \quad (12.2)$$

Even though these two equations involve only the two unknown parameters  $\lambda_1$  and  $A_1$ , their exact solution is not trivial, unless we use an "interpolation trick".

That is, interpolation of the physical data can always be accomplished such that

$$t_{i+1} - t_i = t_{i+2} - t_{i+1} = \dots = t_{i+n} - t_{i+(n-1)} = \tau, \quad (13)$$

where  $\tau$  is a "small time increment". Using the requirement of Eq. 13 with Eqs.

12.1 and 12.2 and generalizing yields

$$\epsilon_{i+1} - \epsilon_i = A_1 e^{-\lambda_1 t_i} (1 - e^{-\lambda_1 \tau}), \quad (14.1)$$

$$\epsilon_{i+2} - \epsilon_{i+1} = A_1 e^{-\lambda_1 t_{i+1}} (1 - e^{-\lambda_1 \tau}) e^{-\lambda_1 \tau}, \quad (14.2)$$

where the left-hand-side of each equation represents the difference between two consecutive experimental strain values. The exact solutions for the model parameters  $\lambda_1$  and  $A_1$  are immediately obtained in terms of the experimental strain data by simultaneously solving Eqs. 14.1 and 14.2:

$$\lambda_1 = (1/\tau) \ln((\epsilon_{i+1} - \epsilon_i)/(\epsilon_{i+2} - \epsilon_{i+1})), \quad (15)$$

$$A_1 = (\epsilon_i \epsilon_{i+2} - \epsilon_{i+1}^2)/(\epsilon_i - 2\epsilon_{i+1} + \epsilon_{i+2}). \quad (16)$$

It should be emphasized that the model parameter solutions of Eqs. 15 and 16 are dependent on only the experimentally obtained strain,  $\epsilon(t)$ , data and can be easily employed by a simple computer program to analyze such data. If indeed this model is appropriate for the analysis of any type of experimental strain data, then Eqs. 10 and 11 can be utilized to predict the mechanical properties. Unfortunately, when this simple model was used with the experimental strain data of Kazarian (1975), no unique values for  $\lambda_1$  and  $A_1$  were realized. A minimization scheme using the exact parameter solutions of Eqs. 15 and 16 could be developed for a more complete analysis of the experimental data; however, the characteristics of the Kazarian (1975) creep response data suggests that a more complex model is required. Since the experimental data graphed in Fig. 1 reveals an elastic response initially, followed by a visco-elastic response, then perhaps a spring placed in series with a Kelvin unit would be appropriate as the next model to investigate.

#### EXACT SOLUTIONS TO THE THREE-PARAMETER-SOLID MODEL

A three-parameter-solid model is illustrated in Fig. 3 as a Kelvin unit and

spring connected in series. Allowing that  $\eta_1$ ,  $E_1$ , and  $E_2$  are related to  $c_1$ ,  $k_1$ , and  $k_2$ , respectively, by equations similar to Eqs. 3 and 4, then the constitutive equation for the total stress applied can be derived in the form

$$\dot{\sigma} + (\eta_1/(E_1 + E_2))\dot{\sigma} = (E_1 E_2/(E_1 + E_2))\dot{\epsilon} + (\eta_1 E_2/(E_1 + E_2))\dot{\epsilon}, \quad (17)$$

where the dot denotes time-differentiation. Eq. 17 can be written in the normalized form as given by Flügge (1975),

$$\sigma + p_1 \dot{\sigma} = q_0 \epsilon + q_1 \dot{\epsilon}, \quad (18)$$

by simply defining

$$p_1 \equiv \eta_1/(E_1 + E_2), \quad (19)$$

$$q_0 \equiv E_1 E_2/(E_1 + E_2), \quad (20)$$

$$q_1 \equiv \eta_1 E_2/(E_1 + E_2). \quad (21)$$

The general solution for the strain of Eq. 18 is again represented by Eq. 6, only now the creep compliance, as given by Flügge (1975), is

$$J(t) = (1/q_0)(1 - e^{-\lambda t}) + (p_1/q_1)e^{-\lambda t}, \quad (22)$$

with  $\lambda = q_0/q_1$ . (23)

From Eqs. 6 and 22, a general form for the strain solution can be expressed by

$$\epsilon(t) = A_1(1 - e^{-\lambda_1 t}) + A_2 e^{-\lambda_1 t}, \quad (24)$$

where the model parameters  $\lambda_1$ ,  $A_1$ , and  $A_2$  are given by

$$\lambda_1 = q_0/q_1, \quad (25)$$

$$A_1 = \sigma_0/q_0, \quad (26)$$

$$A_2 = \sigma_0 p_1/q_1. \quad (27)$$

Using these three equations with Eqs. 19, 20, and 21 allows for the determination of Young's moduli,  $E_1$  and  $E_2$ , and the viscosity coefficient,  $\eta_1$ , in terms of the unknown model parameters:

$$E_1 = \sigma_0/(\lambda_1 - A_2), \quad (28)$$

$$E_2 = \sigma_0/\lambda_2, \quad (29)$$

$$\eta_1 = \sigma_0 / \lambda_1 (A_1 - A_2) = E_1 / \lambda_1. \quad (30)$$

As before, values for the model parameters must be uniquely determined by analyzing the experimentally available strain data.

Using Eq. 24 as the model "data equation", then any three consecutive strain data values obeying the interpolation requirement of Eq. 13 can be represented by

$$\epsilon_1 = A_1(1 - e^{-\lambda_1 t_1}) + A_2 e^{-\lambda_1 t_1}, \quad (31.1)$$

$$\epsilon_{1+1} = A_1(1 - e^{-\lambda_1 t_1 e^{-\lambda_1 \tau}}) + A_2 e^{-\lambda_1 t_1 e^{-\lambda_1 \tau}}, \quad (31.2)$$

$$\epsilon_{1+2} = A_1(1 - e^{-\lambda_1 t_1 e^{-2\lambda_1 \tau}}) + A_2 e^{-\lambda_1 t_1 e^{-2\lambda_1 \tau}}. \quad (31.3)$$

Exact solutions for the three model parameters are now obtainable in terms of the experimental strain data by solving the above three-equations simultaneously. The results for  $\lambda_1$  and  $A_1$  are identical to those given by Eqs. 15 and 16, respectively, and  $A_2$  can be expressed by

$$A_2 = (\epsilon_1 - A_1(1 - e^{-\lambda_1 t_1})) e^{\lambda_1 t_1}. \quad (32)$$

Kazarian and Kaleps (1979) recently analyzed compressive creep phenomena using a three-parameter-solid model described by an equation like Eq. 24. Their results for the Young's moduli and the coefficient of viscosity for a few intervertebral joints are given in Table 1 as  $E_1^*$ ,  $E_2^*$ , and  $\eta_1^*$ . By their analysis scheme, which employed various approximations and optimization schemes, the predicted strain values,  $\epsilon^*(t_i)_{cal}$ , compared favorably with the experimental strain data,  $\epsilon(t_i)_{exp}$ , ranging from about 1% to 15% with about a 5% average.

When the exact parameter solutions developed herein were employed to analyze the Kazarian (1975) data, unique parameter values were obtained for the intervertebral joints considered in Table 1. The results for the Young's moduli and the viscosity coefficient are indicated in Table 1 as  $E_1$ ,  $E_2$ , and  $\eta_1$ , and agree favorably with the results predicted by Kazarian and Kaleps (1979). When the first three experimental strain values (elastic response) are eliminated from the data points associated with each spinal segment, the exact analysis scheme predicts strain values,

$\epsilon(t_1)_{cal}$ , that are within an average of 3.48 % of the experimental data,  $\epsilon(t_1)_{exp}$ , for the twenty spinal segments considered. For each spinal segment considered, the "average of the absolute % error,  $\bar{\epsilon}$ ," is defined by

$$\bar{\epsilon} = (1/N) \sum_{i=1}^N \text{ABS}[(\epsilon(t_1)_{exp} - \epsilon(t_1)_{cal}) / \epsilon(t_1)_{exp} (10^2)], \quad (33)$$

and is observed to be rather small with the exception of test I. D. No. 1 and 16. With these two spinal segments eliminated, the predicted strain values,  $\epsilon(t_1)_{cal}$ , are within an average error of 2.21 % of the experimental data,  $\epsilon(t_1)_{exp}$ , for the remaining eighteen spinal segments. For these intervertebral joints, a graph of strain versus time for  $\epsilon(t_1)_{cal}$  will be, essentially, superimposed on the corresponding graph of  $\epsilon(t_1)_{exp}$ , except for, approximately, the first three strain values.

Inclusion of the first three predicted strain values for each spinal segment results in an average discrepancy of 8% between  $\epsilon(t_1)_{cal}$  and  $\epsilon(t_1)_{exp}$  for the twenty intervertebral joints of Table 1. It should be noted that the exact parameter solutions are very sensitive to the experimental strain values and, thus, critically dependent on the smoothing and interpolation programs employed to reduce the original data. Certainly, an optimization scheme utilizing calculated parameter values could be developed for a more complete analysis of the experimental data, in an attempt to obtain closer agreement between  $\epsilon(t_1)_{cal}$  and  $\epsilon(t_1)_{exp}$  for the first three data points of each intervertebral joint. However, the compressive creep datum analyzed tends to suggest that a slightly more complex model may be required for improved agreement between theoretical and experimental predictions.

#### EXACT PARAMETER SOLUTIONS TO THE TWO-KELVIN-UNIT MODEL

The constitutive equation for the two-Kelvin-unit solid, as illustrated in Fig. 4, can be expressed in the normalized form as

$$\sigma + p_1 \dot{\sigma} = q_0 \epsilon + q_1 \dot{\epsilon} + q_2 \ddot{\epsilon}, \quad (34)$$

where the dot denotes time-differentiation. The stress and strain coefficients of Eq. 34 are related to Young's moduli,  $E_1$  and  $E_2$ , and the viscosity coefficients,  $\eta_1$  and  $\eta_2$ , by the following equations:

$$p_1 = (\eta_1 + \eta_2)/(E_1 + E_2), \quad (35)$$

$$q_0 = E_1 E_2 / (E_1 + E_2), \quad (36)$$

$$q_1 = (\eta_1 E_2 + \eta_2 E_1) / (E_1 + E_2), \quad (37)$$

$$q_2 = \eta_1 \eta_2 / (E_1 + E_2). \quad (38)$$

The general solution to Eq. 34 for the strain, as given by Flügge (1975), is

$$\begin{aligned} \epsilon(t) = \sigma_0 J(t) = & \sigma_0 (1 + p_1 \lambda_1) (1 - e^{-\lambda_1 t}) / q_2 \lambda_1 (\lambda_2 - \lambda_1) \\ & + \sigma_0 (1 + p_1 \lambda_2) (1 - e^{-\lambda_2 t}) / q_2 \lambda_2 (\lambda_1 - \lambda_2), \end{aligned} \quad (39)$$

where  $\lambda_1$  and  $\lambda_2$  are the roots of

$$q_2 \lambda^2 - q_1 \lambda + q_0 = 0. \quad (40)$$

Taking  $\lambda_1$  as the negative root and  $\lambda_2$  as the positive root of Eq. 40, and

$$A_1 = \sigma_0 (1 + p_1 \lambda_1) / q_2 \lambda_1 (\lambda_2 - \lambda_1), \quad (41)$$

$$A_2 = \sigma_0 (1 + p_1 \lambda_2) / q_2 \lambda_2 (\lambda_1 - \lambda_2), \quad (42)$$

then the general solution for the strain behavior can be expressed by

$$\epsilon(t) = A_1 (1 - e^{-\lambda_1 t}) + A_2 (1 - e^{-\lambda_2 t}). \quad (43)$$

Young's moduli and the viscosity coefficients can be related to the four parameters of Eq. 43 by using Eqs. 35, 36, 37, 38, 40, 41, and 42:

$$E_1 = \sigma_0 (\lambda_2 - \lambda_1) / (A_1 (\lambda_1 + \lambda_2) + 2A_2 \lambda_2), \quad (44)$$

$$E_2 = \sigma_0 (\lambda_1 - \lambda_2) / (A_2 (\lambda_1 + \lambda_2) + 2A_1 \lambda_1), \quad (45)$$

$$\eta_1 = E_1 / \lambda_1, \quad (46)$$

$$\eta_2 = E_2 / \lambda_2. \quad (47)$$

Again, values for the four model parameters must be uniquely identified by analyzing the experimentally obtained compressive creep data.

Taking Eq. 43 as representative of the "experimental data", then the four data equations necessary for the exact solution of the model parameters are of the form

$$\epsilon_{i+1} - \epsilon_i = A_1(1 - e^{-\lambda_1 \tau})e^{-\lambda_1 t_i} + A_2(1 - e^{-\lambda_2 \tau})e^{-\lambda_2 t_i}, \quad (48.1)$$

$$\begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \quad \begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \quad \begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix}$$

$$\epsilon_{i+4} - \epsilon_{i+3} = A_1(1 - e^{-\lambda_1 \tau})e^{-\lambda_1 t_{i+3}}e^{-3\lambda_1 \tau} + A_2(1 - e^{-\lambda_2 \tau})e^{-\lambda_2 t_{i+3}}e^{-3\lambda_2 \tau}, \quad (48.4)$$

where the "time interval" requirement of Eq. 13 has been incorporated. Solving Eqs. 48.1 to 48.4 simultaneously and defining

$$E_{i+j} \equiv \epsilon_{i+j} - \epsilon_{i+(j-1)}, \quad j = 1, 2, 3, 4, \quad (49)$$

gives the parameter solutions in the form

$$A_1 = \frac{\epsilon_{i+1}(1 - e^{-\lambda_2 \tau}) - \epsilon_i(1 - e^{-\lambda_2 \tau})e^{-\lambda_2 \tau}}{(1 - e^{-\lambda_1 \tau})e^{-\lambda_1 \tau}(1 - e^{-\lambda_2 \tau}) - (1 - e^{-\lambda_1 \tau})(1 - e^{-\lambda_2 \tau})e^{-\lambda_2 \tau}}, \quad (50)$$

$$A_2 = \frac{\epsilon_i(1 - e^{-\lambda_1 \tau})e^{-\lambda_1 \tau} - \epsilon_{i+1}(1 - e^{-\lambda_1 \tau})}{(1 - e^{-\lambda_1 \tau})e^{-\lambda_1 \tau}(e^{-\lambda_2 \tau}) - (1 - e^{-\lambda_1 \tau})(1 - e^{-\lambda_2 \tau})e^{-\lambda_2 \tau}}, \quad (51)$$

$$0 = (E_{i+2}^2 - E_{i+1}E_{i+3})e^{-2\lambda_1 \tau} + (E_{i+1}E_{i+4} - E_{i+2}E_{i+3})e^{-\lambda_1 \tau} + (E_{i+3}^2 - E_{i+2}E_{i+4}), \quad (52)$$

where  $\lambda_1$  is the negative root and  $\lambda_2$  is the positive root of Eq. 52.

When the compressive creep data of Kazarian (1975) was analyzed by the above parameter equations, no unique set of values for  $\lambda_1$ ,  $\lambda_2$ ,  $A_1$ , and  $A_2$  was obtained. Some calculated values appeared repetitively as the experimental data was tested, but it was not possible to identify a "unique set" of parameter values for any intervertebral joint. As such, use of the two-Kelvin-unit solid as an analytical model was seriously questioned.

#### CONCLUSION

The analytical modelling of compressive creep data by Kelvin solids has been considered. Unique model parameter solutions are accomplished by a mathematical scheme, which has been fully illustrated for the two, three, and four-parameter solids. The first and last models discussed are not capable, by themselves, of predicting observed compressive creep phenomena; however, successful modelling of the experimental data appears possible by using the three-parameter-solid model or

more complex Kelvin chains. More precisely, the five-parameter-solid model is suggested as the next logical model to develop. It is expected to yield a better fit to the experimental compressive creep data for the entire time-domain than the three-parameter-solid model. Exact parameter solutions, of the type discussed herein, have not been obtained for a Kelvin chain consisting of three or more Kelvin elements. It is suggested that any such derivational attempts will be intimidated by the mathematical complexity and uncertainty arising from the algebraic solutions. It should, however, be possible to develop a generalized approximating scheme, utilizing the equations presented herein, which is capable of identifying a unique model for the prediction of compressive creep response. Further, it should be realized that experimental data analyzed by this type of method should be carefully smoothed and interpolated, because of the sensitivity of the equations representing the exact solutions for the model parameters.

The implications of this report are that with improved computer programs pertaining to interpolation, smoothing, and analysis models (development of the five-parameter-solid model), all of Dr. Leon E. Kazarian's experimental data on the load-deflection behavior of intervertebral joints can be quickly, efficiently, and accurately reduced via exact parametric solutions, with associated Young's moduli and viscosity coefficients obtained. Such an accomplishment would represent the most authoritative research results on intervertebral joints to date and should become the primary reference for investigators associated with problems ranging from the man-machine environments to those encountered in the selection of suitable materials for disc replacement.

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#### ACKNOWLEDGMENTS

The author wishes to express his gratitude and appreciation to Mr. Ints Kaleps, 6570 AMRL/BBM, Wright-Patterson Air Force Base, Dayton, Ohio for his assistance and many enlightening discussions. A most appreciative thanks is extended to Dr. Leon E. Kazarian, 6570 AMRL/AMD, Wright-Patterson Air Force Base, Dayton, Ohio for providing the experimental compressive creep data of human intervertebral joints. Lastly a special thanks is extended to Mr. Dennis Likens, Department of Physics, Tuskegee Institute, Tuskegee Institute, Alabama for developing the computer programs necessary for the smoothing and interpolation of the experimental data.

**Figure 1**

A comparison of the experimental compressive creep response for the human T4 - T5 intervertebral joint, by Kazarian, with the predictions of a 5-Kelvin-unit model, by Burns and Kaleps.

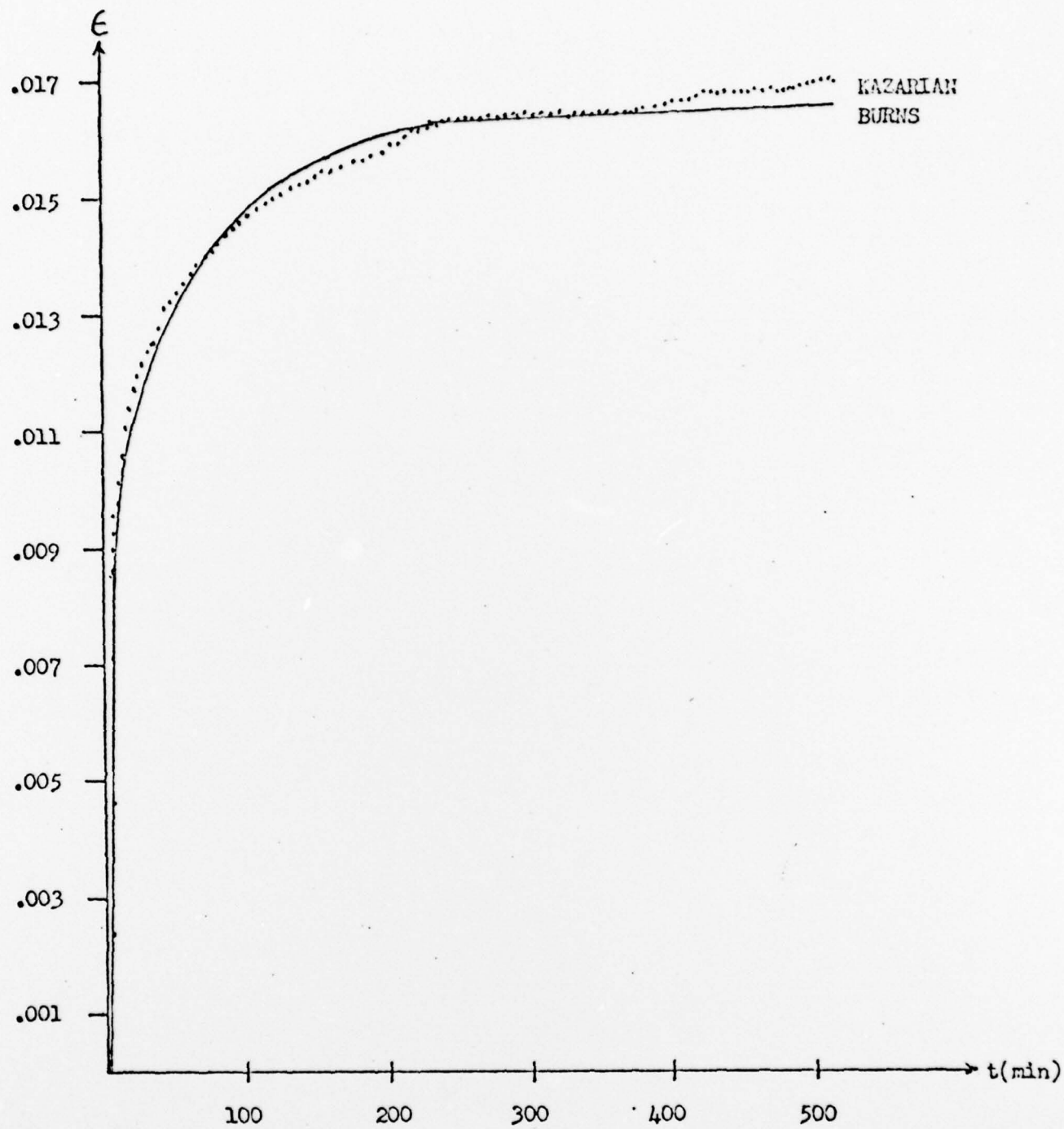


Figure 2

The one-Kelvin-unit solid.

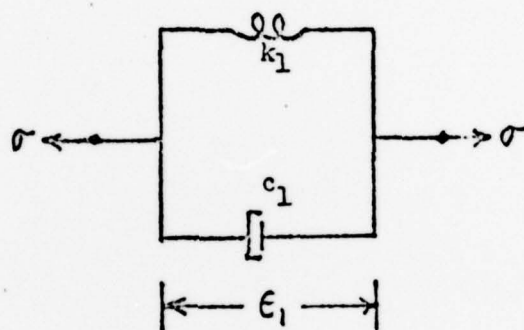


Figure 3

The three-parameter solid.

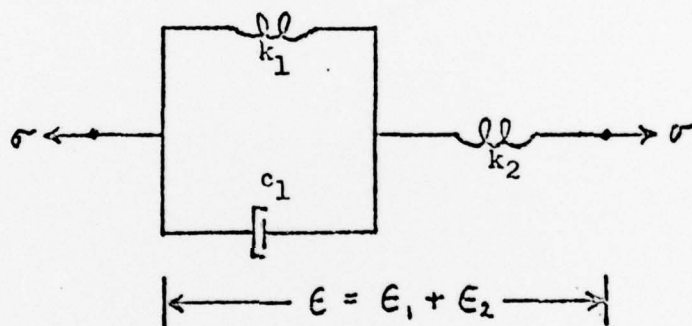


Figure 4

The two-Kelvin-unit solid.

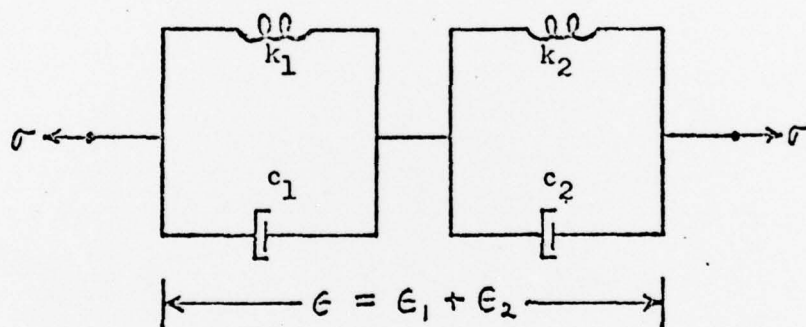


Table 1

A comparison of the Kazarian and Kaleps (1979) Young's moduli ( $E_1^*$ ,  $E_2^*$ ) and viscosity coefficient ( $\eta_1^*$ ) for intervertebral joints is made with those predicted by Burns and Kaleps and represented by  $E_1$ ,  $E_2$ ,  $\eta_1$ , respectively. A comparison of the predicted strain values,  $\epsilon(t_i)_{cal}$ , of the Burns and Kaleps exact analysis scheme with the strain values obtained experimentally,  $\epsilon(t_i)_{exp}$ , by Kazarian (1975) is represented as an "average of the absolute % error", where the first three strain data values for each intervertebral joint has been eliminated.

Test I.D. No.	Spinal Segments	Young's Moduli ( $\times 10^6$ g/cm $^{-2}$ )		Viscosity Coef. ( $\times 10^{12}$ g/cm $^{-s}$ )		Number of Data Points	Average of ABS(% Error)
		$E_1^*$	$E_2^*$	$\eta_1^*$	$\eta_2$		
1	T1 - T2	1.34	3.75	1.16	1.90	106	19.0
2	T2 - T3	2.89	2.33	.583	.497	97	.670
4	T4 - T5	5.27	3.54	3.85	1.58	115	4.45
5	T5 - T6	1.60	3.80	.877	1.31	95	.730
6	T6 - T7	.867	1.65	.625	.879	110	2.16
7	T7 - T8	1.81	2.39	.858	.636	99	2.51
8	T8 - T9	2.72	.670	2.81	1.86	73	2.23
9	T9 - T10	1.43	2.90	1.70	2.52	138	2.43
10	T10 - T11	1.54	3.12	1.87	2.43	131	1.67
16	T2 - T3	.852	1.44	.529	2.70	102	10.8
17	T3 - T4	2.32	6.76	2.08	3.17	115	1.24
18	T4 - T5	1.35	2.59	1.22	1.97	105	3.07
19	T5 - T6	14.0	6.50	1.96	6.13	104	4.46
20	T6 - T7	4.12	3.29	2.86	6.58	75	1.39
21	T7 - T8	1.08	.655	.659	.540	72	2.26
22	T8 - T9	.741	1.26	.585	.868	84	1.57
23	T9 - T10	.926	2.34	.905	1.28	95	3.51
24	T10 - T11	.840	1.39	.654	1.09	76	1.50
25	T11 - T12	1.21	1.44	.986	2.44	81	1.98
26	T12 - L1	1.59	1.57	1.24	2.55	81	1.90

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER <b>AFOSR-TR-79-0795</b>	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) <b>ANALYTICAL MODELLING OF LOAD-DEFLECTION BEHAVIOR OF INTERVERTEBRAL DISCS SUBJECTED TO AXIAL COMPRESSION</b>	5. TYPE OF REPORT & PERIOD COVERED <b>Final research report 01 April 1971-31 May 1979</b>	
7. AUTHOR(s) <b>Marshall L. Burns</b>	6. PERFORMING ORG. REPORT NUMBER	
9. PERFORMING ORGANIZATION NAME AND ADDRESS <b>Tuskegee Institute Department of Physics Tuskegee, Alabama 36088</b>	8. CONTRACT OR GRANT NUMBER(s) <b>AFOSR-78-3578</b>	
11. CONTROLLING OFFICE NAME AND ADDRESS <b>Air Force Office of Scientific Research (NL) Bolling Air Force Base DC 20332</b>	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS <b>61102F 2312D9</b>	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) <b>162312 17D97</b>	12. REPORT DATE <b>31 May 1979</b>	
	13. NUMBER OF PAGES <b>23</b>	
	15. SECURITY CLASS. (of this report) <b>Unclassified</b>	
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE		
16. DISTRIBUTION STATEMENT (of this Report)  <b>Approved for public release; distribution unlimited.</b>		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <p>The analytical modelling of creep response phenomena of intervertebral discs subjected to a constant axial compressive load is attempted by using Kelvin-solid models. A mathematical analysis scheme is proposed for unique model identification wherein exact parameter solutions are developed for the one-Kelvin-unit model, the three-parameter-solid model, and the two-Kelvin-unit model. In addition, a method is presented by which the associated Young's moduli and viscosity coefficients for an identified model are obtainable. Most importantly, unique parameter values are obtained for the three-parameter-</p>		

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411 274

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solid by utilizing exact model parameter solutions on experimental strain,  $E(t)$ , data. This particular model is observed to yield theoretical strain,  $E(t)_{cal}$ , values that are within an average error of 3.48% of the experimentally measured values,  $E(t)_{exp}$ , for different intervertebral discs. Further, mechanical properties of the intervertebral discs are obtained by using the values of the three-parameter-solid model parameters to calculate the associated Young's moduli and viscosity coefficient. The appropriate applications, data limitations, and possible generalizations of this exact analysis scheme are fully discussed, along with suggestions for future investigatory efforts.